Tutorial 1

Microeconomics 3, Part 2

Question 1

Consider a pure exchange economy with $I = 3$ consumers and $L = 3$ goods with the following utility functions and initial endowments:

$$
\begin{aligned}
u_1(x_{11}, x_{21}, x_{31}) &= \min\{x_{11}, x_{21}\} \qquad \text{with} \qquad \boldsymbol{\omega}_1 = (\omega_{11}, 0, 0) \\
u_2(x_{12}, x_{22}, x_{32}) &= \min\{x_{22}, x_{32}\} \qquad \text{with} \qquad \boldsymbol{\omega}_2 = (0, \omega_{22}, 0) \\
u_3(x_{13}, x_{23}, x_{33}) &= \min\{x_{13}, x_{33}\} \qquad \text{with} \qquad \boldsymbol{\omega}_3 = (0, 0, \omega_{33})
\end{aligned}
$$

The consumption set, X_i, is $\mathbb{R}^3_+$ for all consumers. There is a single firm whose only production technology is free disposal ($Y_1 = -\mathbb{R}^3_+$). Each consumer owns a one-third share in the profits of this firm ($\theta_{i1} = \frac{1}{3}$ for all $i = 1, 2, 3$).

(i) Suppose $\omega_{11} = 1$, $\omega_{22} = \frac{1}{2}$ and $\omega_{33} = 1$. Solve for a Walrasian equilibrium in this economy and provide the normalized price vector and equilibrium allocation for each consumer and the firm. You may normalize the price of good 1 to 1.

(ii) Suppose instead $\omega_{11} = 1$, $\omega_{22} = 2$ and $\omega_{33} = 1$. Is there a Walrasian equilibrium in this economy? If so, solve for the price vector and allocation as in (i). If not, show that there cannot be one.

Question 2

There is an economy with two goods, two consumers and one firm. One of the goods (good x) is a public good and the other good (good m) is a rival and excludable composite commodity. Each consumer's utility function is:

$$
\begin{aligned}
u_1(x, m_1) &= \log(x) + m_1 \\
u_2(x, m_2) &= 2\log(x) + m_2
\end{aligned}
$$

The firm's production technology is as follows: one unit of good x can be produced from two units of good m. Each consumer is endowed with 0 units of good x and 5 units of good m, so $\boldsymbol{\omega}_i = (0, 5)$ for $i = 1, 2$. Each consumer has an equal share in the profits of the firm. Normalize the price of good m to 1 and use p for the price of good x.

(i) Show that the Pareto optimal quantity of the public good is $x^\circ = \frac{3}{2}$.

(ii) What is the price p and allocation $(x^\star, m_1^\star, m_2^\star)$ in the competitive equilibrium of this economy?

(iii) If a per-unit subsidy is placed on good x, what does the subsidy need to be in order to achieve the Pareto optimal quantity, x°?

(iv) Suppose the subsidy you found in (iii) needs to be financed by a per-unit tax, t, on the consumption of good m. What should the tax on good m be to balance the government's budget (i.e. the total revenue from the tax equals the total cost of the subsidy)?

(v) *Note: This question is unrelated to (iii) and (iv).* Suppose the government taxed individuals based on "not contributing their fair share" of the public good. The tax paid by a consumer who contributes x_i to the public good is:

$$\begin{cases} t\left(\frac{x^\circ}{2} - x_i\right) & \text{if } x_i < \frac{x^\circ}{2} \\ 0 & \text{if } x_i \geq \frac{x^\circ}{2} \end{cases}$$

where $x^\circ = \frac{3}{2}$ and $t = \frac{3}{2}$. Consumer i is taxed if they contribute less than half of the Pareto optimal quantity, x°. However, if they contribute at least half, they are not taxed.

Show that with $t = \frac{3}{2}$ there is an equilibrium in which both consumers choose $x_i = \frac{x^\circ}{2} = \frac{3}{4}$, and the Pareto optimal quantity of the public good is provided.

Question 3

In a pure exchange economy there are two consumers and two goods. The two consumers are are two individuals who share an apartment. The two goods are money and cigarettes. Consumer 1 enjoys smoking cigarettes but Consumer 2 obtains disutility when Consumer 1 smokes.

Each consumer's utility is:

$$u_1(x_{11}, x_{21}) = \sqrt{x_{11}x_{21}}$$
$$u_2(x_{12}, x_{21}) = \sqrt{x_{12}(20 - x_{21})}$$

where $x_{\ell i}$ is i's consumption of the ℓth good, where good 1 is money and good 2 is cigarettes. Notice that if Consumer 1 consumes more cigarettes, Consumer 2's utility decreases. If Consumer 2 consumes cigarettes, neither consumer's utility is affected (think of Consumer 2 as freely destroying any cigarettes that Consumer 1 doesn't smoke).

The total endowment of money in the economy is €100 and the total endowment of cigarettes is 20. The endowment of money is split equally among consumers (€50 each). The distribution of the endowment of cigarettes can vary question-by-question.

In parts (i)-(ii), there is a market for both goods. In this market, the price per cigarette is p and the price of money is normalized to 1.

Hint: Since there are 20 cigarettes in total in the economy, you could consider Consumer 2's utility function as $u_2(x_{12}, x_{22}) = \sqrt{x_{12}x_{22}}$ where $x_{22} = 20 - x_{21}$ is the number of cigarettes *not smoked* by consumer 1.

(i) Suppose Consumer 1 owns the full endowment of cigarettes and there is a market for both goods. What is the equilibrium price and allocation? How many cigarettes are smoked in total?

(ii) Now suppose *Consumer 2* owns the full endowment of cigarettes. What is the equilibrium price per cigarette and allocation?

(iii) Find the full set of Pareto allocations for this economy.

(iv) Consumer 1 owns the full endowment of cigarettes like in part (i), but there is no longer a market for cigarettes. Show that if Consumer 2 can make a take-it-or-leave-it offer to Consumer 1 to reduce cigarette consumption, that the optimal offer is $T = 50\left(\sqrt{2} - 1\right) \approx$ €20.71 to reduce consumption to $\frac{20}{\sqrt{2}} \approx 14.14$ cigarettes.

Question 4

Consider an economy with 2 goods, a consumption good x and a composite commodity m. There are 3 consumers with the following utility functions overs these 2 goods:

$$u_1(m_1, x_1) = m_1 + 4\log(x_1)$$
$$u_2(m_2, x_2) = m_2 + 8\log(x_2)$$
$$u_3(m_3, x_3) = m_3 + 12\log(x_3)$$

where $\log(\cdot)$ is the natural logarithm. Consumers can only consume a nonnegative quantity of the good x. There are two firms that produce the consumption good (output q) from the composite commodity (input z). Their production sets are:

$$Y_1 = \left\{(-z_1, q_1) : q_1 \geq 0 \text{ and } z_1 \geq \frac{1}{2}q_1^2\right\}$$
$$Y_2 = \left\{(-z_2, q_2) : q_2 \geq 0 \text{ and } z_2 \geq q_2^2\right\}$$

The total endowment of the composite commodity in the economy is $\bar{\omega}_m = 24$. There is no initial endowment of the consumption good.

(i) Formally specify the social planner's problem assuming the planner maximizes the unweighted sum of the consumers' utilities. Solve for the Pareto optimal allocation of x_i across consumers and q_j across firms.

(ii) Use (i) to specify the Pareto frontier.

(iii) Assume the price of the consumption good is p and the price of the composite commodity is normalized to 1. The distribution of the initial endowment of the composite commodity is $(\omega_{m1}, \omega_{m2}, \omega_{m3}) = (8, 8, 8)$. There is no initial endowment of the consumption good. Each consumer owns an equal share in the profits of each firm. Solve for the equilibrium price p and allocation of both goods (for both consumers and firms).

(iv) Show that your answer to (iii) is on the Pareto frontier in (ii).